SOUNDING WAYS INTO MATHEMATICS

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Abstract

Music and mathematics are both sign systems, which overlap in many ways. The project "Sounding ways into mathematics" aims to direct attention to the interconnections between music and mathematics, and to promoting creative didactic ways to combine them in the classroom. The underlying theory of semiotic bundles describes how communication and learning depends on simultaneous use of different sign systems. The project is part of the European Music Portfolio. Contributors from teacher education institutions in seven European countries cooperatively develop materials. Examples are collected in a teacher handbook which also provides theoretical background on the music-math activities for the classroom. In its last phase, the project conducts teacher education courses in several European countries with regards to the practice of combining music and mathematics.

KEY WORDS: Mathematics, music, semiotic system, experiences, transversal learning.

Introduction[26]

Music and mathematics share an odd character: many people believe that they are not good at one or the other. The project "European Music Portfolio – Sounding Ways into Mathematics" (EMP-Maths) aims towards a different understanding. Everyone can sing and make music, and everyone can do mathematics.

[26] This text is an excerpt from the EMP-Maths Teacher Handbook (Mall, Spychiger, Vogel, & Zerlik, 2016)

Both topics are integral parts of our life and society. What needs to be improved is our ability to give students opportunities to like them.

The project "Sounding Ways into Mathematics" is a collaboration of seven teacher training institutes in order to develop a CPD (continuous professional development) course, that trains teachers in transversal learning techniques, focused on music and mathematics. It is actually in its final phase and most of its publications are ready for download from the projects website: maths.emportfolio.eu. The project was funded by the European Union through the Lifelong Learning Programme.

This article highlights some of the core concepts behind the project: pattern recognition, music and mathematics as sign systems, perception and action and making experiences. Finally we want to introduce two activities developed for use in the classroom, that are part of the CPD courses provided by the EMP-Maths consortium.

Pattern recognition. Pattern recognition is, first of all, paying attention to the connecting pattern (Bateson, 2002, p. 16) and therefore is a basic human activity that is bound to awareness. Some theories claim that attention is rhythmically organised (Auhagen, 2008, p. 444). Attention to, and awareness of, connecting mechanisms can be observed in children very frequently, and they often include expressions of happiness. Rope skipping, jumping muddy puddles, and making rhythmic noises with sticks on fences are happy childhood activities. The human capability for rhythmic synchronisation, as well as pattern recognition, begins in early childhood and seems to be encouraged by dandling babies on the knees (Fischinger & Kopiez, 2008, p. 459).

Humans have the capacity to follow rhythmic patterns from the first. Experiments with newborn babies prove this very fact in that they are able to differentiate between rhythmic and non-rhythmic clicks (Gembris, 1998, p. 403f.). Even early on, while floating in the womb of the mother, their leg movements show patterns of tempo, which are in time with the mother's heartbeat (Gruhn, 2005, p. 126). These early rhythmic musical abilities have in common the baby's capability of recognising patterns and tuning into them, or, as Björn Merker has put it, can "entrain to a repetitive beat" (Merker, 2000, p. 59). Later, entrainment is obvious in countless activities, mostly through play; for example, with a ball in groups, in increasingly complex activities – such as when accompanying rhythmic language patterns and rhymes with movement – and in singing activities.

Another important aspect of pattern recognition is classification or *chunking* (Jourdain, 2001, p. 163). Chunks are small packages of information that we can handle as one unit. Chunks are treated hierarchically. From small chunks, bigger ones are created. From those, further and bigger chunks are built, and so on. As a matter of fact, we create patterns in order to chunk. Listening to a constant sequence of similar tones leads to building groups of two or three (Auhagen, 2008, p. 439), and therefore building (rhythmical) patterns. Alikeness, nearness and similar behaviour are all features that enable mental pattern recognition. Not only can we recognise patterns, but we also construct them and give meaning to them.

For example, the significance of chunks for the interaction with patterns (Vogel, 2005, p. 446) becomes important during the exploration of geometrical patterns. "During the exploration it is important that the base elements or units of the phenomenon are found" (ibid.). Only the identification of these base units enables the mathematical analysis of complex ornaments and clarifies the fascination of mathematics. Composers use this capacity in order to write polyphonic pieces for monophonic instruments. They group tones in a way that means our ear and mind "hear" two or more different voices.

Pattern recognition is an important task for hearing sounds (Bharucha & Mencl, W. Einar, 1996). Recognising the sounds of instruments and octave equivalence is a pattern recognition task, as is our ability to categorise tones C, D, E, F, G, A and B as a major scale and recognise the same melody when it is played in different keys. This shows "that pitch and key can serve to gate spectra into pitch-invariant representations" (Bharucha & Mencl, W. Einar, 1996, p. 149). Bharucha et al. suggest that "western listeners seem to have a highly elaborated representation of keys and their relationships" (ibid., p. 148). Several studies show that this is also important for sight-singing ability (Fine, Berry, & Rosner, 2006). This is especially the case with the ability to predict following tones in sequences; this ability is better when these tones are part of tonal melodies or well-known patterns.

The need for pattern recognition and synchronisation is rooted in nature. Small animals that hunt bigger ones synchronise their steps in order to catch them (Fischinger & Kopiez, 2008, p. 460), and chimpanzees synchronise their voices in order to increase the distance at which they can be heard (Merker, 2000).

The childhood games mentioned above, as well as activities such as rope skipping, jumping muddy puddles and dancing, are occasions to practise coordination and pattern recognition (Spychiger, 2015a).

Pattern recognition and grouping enable us to do things simultaneously: marching, rowing, clapping and playing symphonies. Doing things together (and letting others know about it) enforces the group, attracts females and keeps enemies away; this is true at a recreation place's campfire as well as deep in the jungle, where chimpanzees do exactly the same thing (Merker, 2000). When things are done in complete synchronisation, they are louder and more effective. Pattern recognition therefore is at the core of the shared characteristics in mathematical and musical activities.

In all kinds of human activities, people show how they are not only capable of recognising patterns, but also of creating and producing them. This takes us to the semiotic function circle model, which offers the integration of those two aspects in human behaviour – perception and action, as explained in the next chapter (figure 1).

Music and mathematics as sign systems

Early semiotic theories described communication as a linear process where information was directly transferred from one person to the other. Charles S. Pearce as an alternative developed the triadic classification of the semiotic process, with the subject–object–sign system. Still, this system defines communication processes. Music, therefore, was not seen as a sign system, first of all because it "has not an object to signify" (Spychiger, 2001, p. 55), and also because it is not the basis of a valid communication process.

Alfred Lang (1993) developed a semiotic model based on how a person relates to the world. It looks at sign systems as the basis of human perception and action in an ongoing way, as shown in the semiotic function circle (figure 1).[27] This approach denies the need for a distinction between subject and object, and instead distinguishes between processes that "take place *within* the person and […] *outside* of the person" (Spychiger, 2001, p. 57), using the terms "presentant" (in the place of the object) and"'interpretant" (in the place of the subject). Musical mental processes, then, take place in a circular way; a *musical perception* ('IntrO', what comes in) leads to a *musical experience* ('IntrA', what takes place within the person) that can evoke *musical production* ('ExtrO', what goes out from the person into the world). "These musical actions then manifest themselves outside of a person: as *musical culture*" (which is 'ExtrA', ibid., p. 58). This point closes the circle, which then creates new opportunities of musical perception again (as the arrows show in figure 1).

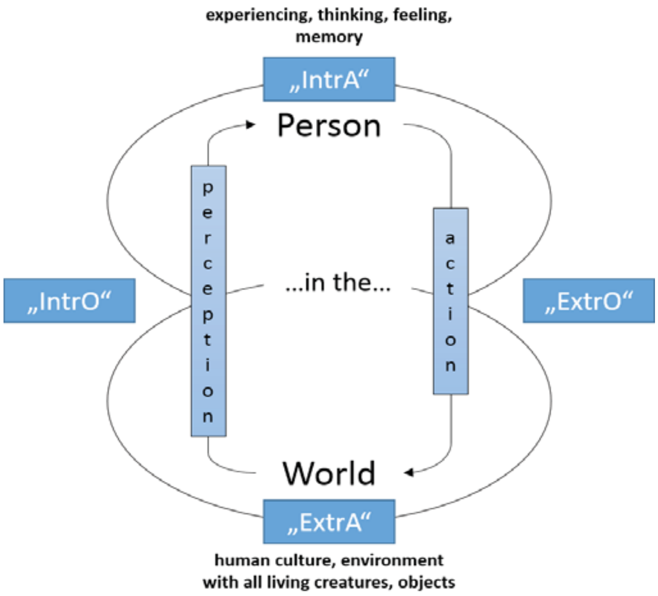

Figure 1. General psychological model of the person–world relationship. Semiotic function circle (according to Spychiger, 2001).

[27] Summarised in Spychiger (2001, p. 56).

Understanding music as an independent sign system again makes it possible for us to compare this system with other systems, e.g. mathematical ones, without neglecting the independent reason for music. We can search for and find musical principles that can be explained mathematically. Music is full of symmetries, and notation is a system with mathematical accuracy.

With the linear thinking left behind, many more sign systems were possible and more communicative aspects (gestures, mimics) could be seen as independent sign systems. In actual communication, all these systems interact and build a semiotic bundle (Arzarello, 2015). In modern educational theories for teaching and learning, these bundles play an important role, because, with this approach, teaching processes and interaction in the classroom can be described much more precisely.

With music and mathematics being proper sign systems and the theory of semiotic bundles, interdisciplinary projects may gain new meaning. Just as gestures and mimicry complement aural communication, mathematics can be used to explain music, and vice versa.

Although we do not believe that music is a mathematical system and vice versa, there are numerous connections between both worlds (Bamberger, 2010; Brüning, 2003; Christmann, 2011; Lorenz, 2003). With the concept of semiotic bundles, we want to develop creative learning environments[28] to bring together several semiotic systems.

Perception and action

Perception and action are central elements in the semiotic function circle, which describes person–world interaction: perception brings information into the person while, through the action, the person interacts with the world. Inside the person, perception creates knowledge and action creates culture in the world (see figure 1).

In music education, this unity was not always obvious, as music education for many years was nothing more than singing lessons. Only in the 1920s (in Germany), with the reform of Leo Kestenberg, did music education develop to find a place in the scientific community, becoming a concern in teacher education as well as in schools.

Still, action and perception were concurrent aspects in the philosophy of music education for a long time (Spychiger, 1997). Especially well known is the discussion between Bennett Reimer and David Elliott. Reimer, on the one hand, claimed that "school music programs exist [only] to provide communities with a variety of social services" (Reimer, 1989, p. 24). As a consequence, he wanted to strengthen perception of music in the curriculum, and provide students with aesthetic experiences while learning about the great musical artworks.

David Elliott, on the other side, criticised the prevalence of classical music in the curriculum and corresponding teaching concepts, especially the lack of acceptance of affective elements (Elliott, 1987). Together with Christopher Small, Elliott supports music making – musicking – as a central element in the classroom (Elliott & Silverman, 2014; Small, 1998). The semiotic function circle explains the importance of both elements – action and perception – for music education (Spychiger, 1997), as has been shown in the general model for human life as a whole.

In modern mathematics education, the interplay of perception and action also gets more and more important as students use action and perception cycles to develop mathematical understanding. A central element of mathematics is the *close look*. The identification of patterns and their translation into a sign system is a central mathematical task. Repetitions and, therefore, regularities can be found by observing the written symbols. These regularities are the basis for mathematical insights. During the lessons, students reconstruct this approach. Mathematical tasks serve as stimuli for activities performed on paper. With the analyses of these activities, based on perception, regularities are found and transformed into awareness.

The discovery of mathematical aspects in everyday phenomena works in the same way. A modelling process transfers central aspects of the real situation into a realistic model that contains the central structural elements of the real situation. This is the foundation of a mathematical model. Action forces children to discover mathematical regularities and structural principles in the classroom. Here, making mathematics would be the corresponding concept to musicking – making music. All the activities shown in the *EMP-Maths*

[28] Such learning environments are shown in more detail in chapter 4.1 of the EMP-Maths Teacher Handbook (Mall, Spychiger, Vogel, & Zerlik, 2016, p. 19)

Teacher Handbook (Mall, Spychiger, Vogel, & Zerlik, 2016) combine elements of action and perception to open minds and encourage emotions.

Making experiences

The activities developed in this project are meant to open the learning environments in which mathematical and musical experiences can be formed. Musical and mathematical content are merged. They should provide insights for both topics. Interdisciplinary learning environments frame this content differently and therefore allow experiences that are not possible in subject-oriented learning situations.

As John Dewey (1925; 1980/1934) understood it, "experience" is an "interactive, comprehensive event that contains not only cognitive but also affective, emotional and aesthetic components" (Neubert, 2008, pp. 234–235). We follow his approach in not putting cognition at the centre of learning, but instead – "experience". First, before reflection and thinking, we are immersed in feelings, aesthetic perception, and current situational impressions (ibid., p. 235).

A sequence is taken intuitively out of the "stream of events" (Spychiger, 2015b, p. 111), and is turned into an experience from this emphasis. Real experience is a temporally limited unit with emotional quality, descriptive character and nameable content: "Those things, of which we say in recalling them, 'that was an experience' […] – a quarrel with one who was once an intimate, a catastrophe finally averted by a heir's breath […], that meal in a Paris restaurant […]" (Dewey, 1980/1934, p. 37). According to Dewey, experiences are additionally marked by their communicative character. Through interaction, people can participate in the experiences of others and potentially gain other perspectives on their own experiences (cf. Neubert, 2008, p. 238).

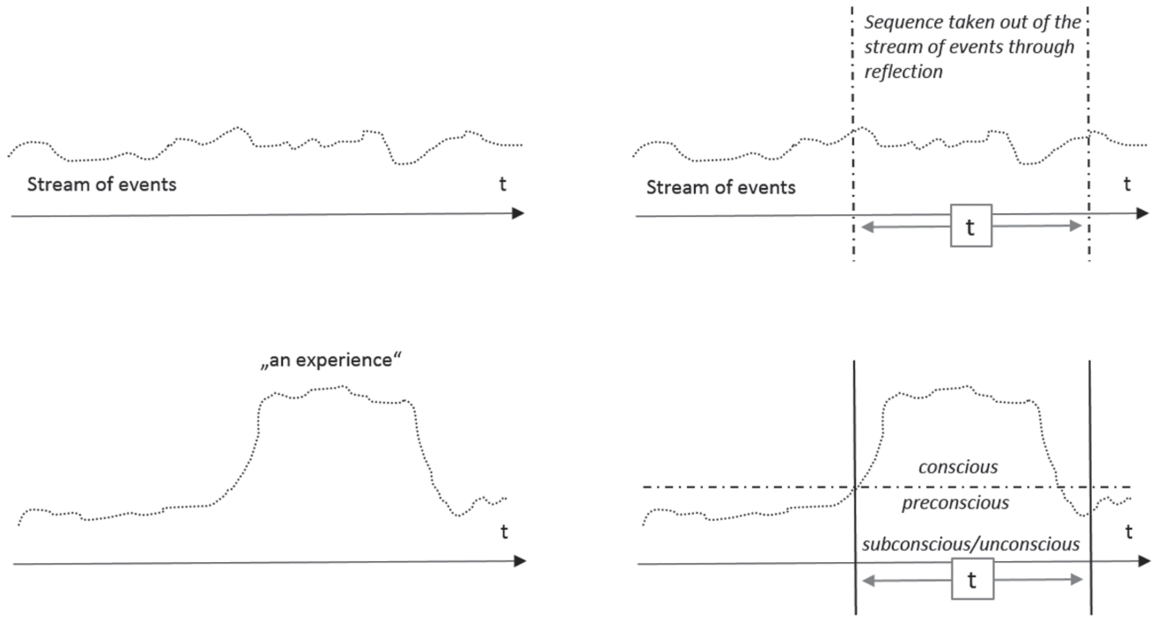

Figure 2. Visualizing the concept of experience related to John Dewey (1980/34) (Spychiger, 2015, p. 112).

Against the background of Dewey, the EMP-Maths activities provide learning environments where experiences are possible.

The EMP-Maths activities are developed to provoke entangled mathematical and musical events (see figure 2). The situational base for the participants to have new experiences with mathematics and music, or both, is created through the focus on a selection of singular events, for example through reflection and group discussions. This approach can help to change the unconscious image of mathematics and music through experiences in the EMP-Maths activities.

Activities

The presentation of activities in all publications follows the concept of didactic design patterns (Vogel & Wippermann, 2011). Design patterns describe general solutions for repeating problems in standardised ways through formal structures. The formal structure in the EMP-M project consists of the parts *overview*, *preparatory deliberations*, *implementation* and *variations*, of which *implementation* describes the actual content together with a standard approach. The use of design patterns makes not only the process of developing activities easier, but also helps teachers to easily access all necessary information.

Clapping the lowest common multiple of 2, 3 and 5

This activity uses body percussion elements to let students discover the lowest common multiple of 2, 3 and 5. First, the students learn body percussion patterns, each related to the numbers 2, 3 and 5. All patterns end with a clap of the hands and use different body timbres (snap fingers, slap thighs, pound chest, stomp foot), so they can more easily heard separately. In different steps, students learn to play their body percussion pattern while counting to 30. Then different groups perform their patterns simultaneously, first 2 and 3, until all three groups play together, still counting to 30.

Clapping and counting shows the multiples of each number through the clap of the hand. Every time, two groups have this sound together, a common multiple is found, with the first appearance being the lowest common multiple. Finally, reaching 30, all three groups clap together, showing the lowest common multiple of 2, 3 and 5.

Twinkle, twinkle little star

This song offers multiple opportunities to introduce *patterns*, *symmetries* and *reflections* (using a mirror) to students. Depending on what musical element students focus, they will discover different patterns. It also provides students with the possibility to express themselves using their voice.

Students start this activity by singing the song several times. They then find their own way to notate the song, following ups and downs. Each of these steps is accompanied by questions about patterns and symmetries. Step by step, students will discover melodic and rhythmic patterns, build reflections using mirrors and comparing these reflections with their own drawing. Finally, they use motifs of the song in order to create their own patterns and learn to sing them.

This activity is not especially bound to the given song, but with many available versions it is a good starting point. And in the end, students can listen to the variations to this song, composed by W. A. Mozart.

Conclusion

The project 'EMP-Maths: Sounding Ways into Mathematics' aims to highlight the connectedness of music and mathematics in every day live through classroom activities that are short and simple. Music and mathematics play the role of equal partners in a modern interdisciplinary teaching approach.

Teachers are encouraged to join the newly developed CPD courses in order to implement the ideas in the classroom. More information, as well as further activities and an online collaboration platform can be accessed through the project's webpage: http://maths.emportfolio.eu.

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